Journal of Food Engineering 293 (2021) 110372

Contents lists available at ScienceDirect

Journal of Food Engineering

journal homepage: http://www.elsevier.com/locate/jfoodeng

Analysis of random variability in Tortilla shells baking

Rosalina Iribe-Salazar [a], José Caro-Corrales [a,*], Yessica Vázquez-López [b]

[a] Posgrado en Ciencia y Tecnología de Alimentos, Universidad Autónoma de Sinaloa, Avenida de Las Américas y Josefa Ortiz, C.P. 80013, Culiacán, Sinaloa, Mexico
[b] Posgrado en Ciencias Agropecuarias (FMVZ-UAS), Universidad Autónoma de Sinaloa, Blvd. San Ángel, Fracc. San Benito 3886, C.P. 80260, Culiacán, Sinaloa, Mexico

ARTICLE INFO

Keywords:
Baked products
Moisture
Mathematical modeling
Variability

ABSTRACT

Baking of corn tortilla shells can be considered as a random phenomenon. Dispersion in tortilla shell baking was estimated. The solution to Fick's law of diffusion for transient mass transfer was used as the basis for the deterministic model. A Theoretical Probabilistic model was applied to predict the variability in the moisture content. The mean and standard deviation for initial and equilibrium moisture content, half-thickness, and effective diffusivity were considered as the input random variables. Results given by the Theoretical Probabilistic model were validated with Monte Carlo simulations and experiments. The highest variability in the moisture content (0.0165 kg water/kg d. s.) occurred at 190 s. The mean and standard deviation estimated by the Theoretical Probabilistic model satisfactorily fit those obtained with Monte Carlo ($R^2 = 0.9998$ and 0.9950) and experiments ($R^2 = 0.9998$ and 0.9800), for the entire process. A sensitivity analysis indicated equilibrium moisture content (98.53%) caused the greatest dispersion in the final moisture content. This study will allow to suggest baking improvement strategies.

1. Introduction

In Mexico, corn tortilla represents the primary essential food; corn and tortilla chips are the most significant salty snacks (Vázquez-Carrillo et al., 2015). Approximately, 95% of Mexicans consume "tortilla", as it is an important daily source of energy (~50–70%) and protein (~50%) (Martínez-Velasco et al., 2018). Corn snacks are obtained by frying the dough and contain more fat than tortilla snacks. To obtain tortilla snacks, they are baked and then fried, which makes them absorb less fat, acquire a firmer texture, and a stronger alkaline flavor than corn snacks (Kawas and Moreira, 2001). However, after frying, these products retain up to 40% fat and, on the other hand, nowadays, health-conscious consumers demand low-fat snack foods (Serna-Saldivar, 2016). Baked corn tortilla shells known as *tostadas horneadas* are increasing their popularity due to their low-fat content and energy content (Palazoglu et al., 2010).

During baking, a wide variety of chemical, rheological, and structural changes occur inside the food product. As these changes take place, the concentration of the food components in the products, mainly the moisture content, has a significant impact on the final properties of the food; therefore, it is relevant to know the variation in moisture content between baked corn tortilla shells as the baking process progresses (Thorvaldsson and Janestad, 1999). One of the main goals in the baking of corn tortillas is that a specific uniform moisture content is reached at the end of the process; therefore, wide variations in this parameter are unwanted, since it causes poor product quality and high processing costs. However, in practice, variability in moisture content is inevitable and arises from the intrinsic heterogeneity of the food and the stochastic nature of the baking process. In particular, a distribution in the initial moisture content of the food material, variability in the properties of the food due to its biological origin, and random fluctuations in both, the geometry of the processing equipment and the conditions of the baking fluid contribute to the uncertainty in results (Cronin and Kearney, 1998).

Reducing the variability in the moisture content is of great relevance in the mass transfer processes that are carried out in a non-steady state. Diminishing variability requires more accurate knowledge and control of the average levels of moisture content in corn tortilla shells throughout the baking process. Traditionally, the removal of moisture content from food and the associated impact on quality have been analyzed using empirical or phenomenological deterministic models (Cronin and Kearney, 1998). A different approach is to use a probabilistic solution technique such as an analysis using a Theoretical Probabilistic model or a numerical solution procedure such as the Monte Carlo method. These methods are used to estimate the variability in the response variables from the uncertainty in the model input variables. The Monte Carlo method uses a probabilistic sampling procedure, which provides an effective approach to propagation and uncertainty analysis

* Corresponding author.
E-mail address: josecaro@uas.edu.mx (J. Caro-Corrales).

https://doi.org/10.1016/j.jfoodeng.2020.110372
Received 9 August 2020; Received in revised form 19 September 2020; Accepted 10 October 2020
Available online 16 October 2020
0260-8774/

(Helton and Davis, 2003). Feyissa et al. (2012) analyzed the uncertainty and sensitivity using a heat and mass transfer model in a contact baking process and applied the Monte Carlo method to propagate the uncertainty in the model input parameters. Although the Monte Carlo technique is a useful numerical tool it can also be complicated to use due to the large number of trials that must be performed to generate a statistically reliable output variable (Caro-Corrales et al., 2002).

Previous studies have shown that the variability in biscuit surface temperature or the center temperature of discrete solid products can be defined by theoretical probabilistic expressions (Caro-Corrales et al., 2002; Cronin et al., 2000). These expressions are based on the theory of functions of random variables and allow obtaining both, the mean and its variance at any temporal point in the process. An advantage of analytical expressions for product variability is that these equations can potentially be used as the basis of study for process improvement. Therefore, these studies allow us to assess how well these expressions work when applied to food processing. In the present research, the initial (X_0) and equilibrium (X_∞) moisture content, as well as the half-thickness (L), and effective diffusivity (D_e) of the tortilla are considered as input random variables; and the average ($\overline{X}$) moisture content of the tortilla at any time is taken as the output random variable. The results of the present study can be applied to mass transfer operations other than baking and in products that can be represented as an infinite plate, considering the corresponding specific parameters (X_0, L, D_e, and X_∞) of the product under study. The predictions obtained with the Theoretical Probabilistic model can be compared with those of the Monte Carlo method and with experimental results.

A parameter study using a Theoretical Probabilistic model or the Monte Carlo method allows analyzing the influence of these parameters on the variability of moisture content and proposing strategies that decrease this variation. This can lead to an improvement in the quality of the product and, therefore, a decrease in the amount of defective products. In addition, an optimization of the distribution in the average moisture content between baked corn tortilla shells is achieved at the end of the process and allows to reduce the processing costs, which will benefit the producer and exporter of this product. Therefore, the objective of this study was to predict the random variability and determine the influence of input variable dispersion on the average moisture content between corn tortilla shells (*Zea mays* L.) during the hot air baking process using a Theoretical Probabilistic Model and to validate these predictions with the Monte Carlo method and with experimental results.

2. Materials and methods

2.1. Preparation and baking of corn tortilla shells

Tortillas were set up from customary nixtamalized corn flour, which was blended in the proportion 1:1 with water in an industrial blender (Tecnomaiz, M-40, Mexico) for 120 s to form the dough with a moisture of 50%. The thickness of dough was regulated in a roller machine (Rodotec, RT-100 T, Mexico) and plain shells were cooked at 270 °C for 30 s. Corn tortillas had nominal diameter and thickness of 14 cm and 1 mm, respectively. Selection depended on criteria of homogeneity in diameter, thickness, and mass. Baking of tortillas to produce low-fat baked shells was completed in a commercial electric oven (Oster, 6081–013, Mexico) at 180 °C.

2.2. Mass transfer kinetics during baking

The mass of tortilla was recorded every 5 s with an electronic scale (Sartorius, TE1502S, Germany). The average moisture content ($\overline{X}$, kg water/kg dry solid) was calculated with $\overline{X} = (m - m_{ss})/m_{ss}$ where m is the mass of the sample (kg) and m_{ss} is the mass of dry solid (kg d. s.). The moisture content at the commencement of the process was determined by the AOAC (2012) method. The equilibrium moisture content was determined when no changes were observed in the average moisture content (Geankoplis, 2013). The thickness ($2L$) of the sample was measured with a digital caliper (Inzice, CD-8″C, Mexico). Thickness was measured at the beginning of the process and assumed constant. For the baking of corn tortillas, 40 replicates were carried out to determine the mean and standard deviation of the initial and equilibrium moisture content.

2.3. Theoretical modeling

2.3.1. Deterministic baking model

Studies involving drying or baking of corn tortillas have shown that there is no evidence of a constant rate period (Xu and Kerr, 2012). Generally, molecular diffusion determines the moisture transfer rate during the falling rate period (Seth and Sarkar, 2004). A relatively simple representation was used for a single tortilla, considering it as an infinite plate undergoing hot air baking. A hot airflow of 0.5 m/s was used parallel to the upper and lower surfaces of the tortilla. During baking of tortilla, the diffusion coefficient increases due to temperature and structural factors, such as pore size and number distribution, and pore expansion by pressurization of air and water vapor. These changes in the diffusion coefficient can be incorporated through Fick's 2nd law of diffusion considering an effective diffusivity with a quadratic function of time as baking proceeds,

$$\frac{\partial X}{\partial t} = \left(D_0 + D_1 t + D_2 t^2\right)\frac{\partial^2 X}{\partial x^2} \tag{1}$$

where X is moisture content (kg water/kg d. s.), t is baking time (s) and x is the spatial dimension (m). The effective diffusivity in Equation (1) is $D_e = D_0 + D_1 t + D_2 t^2$. The initial effective diffusivity is indicated by D_0 (m^2/s); the rate of change of diffusivity varies linearly ($D_e' = D_1 + 2D_2 t$) as the baking progresses and the initial rate of change is D_1 (m^2/s^2). The increase ($D_2 > 0$) or decrease ($D_2 < 0$) in the rate of change for the diffusion coefficient ($D_e'' = 2D_2$) depends on D_2 (m^2/s^3), which is related to pore size and number distribution around a matrix of partially gelatinized starch and, mainly, to the expansion of pores.

The x coordinate is measured from the center of the plate, which is $2L$ thick. The initial moisture content X_0 was considered uniform and the plate was suddenly immersed in hot air with constant humidity X_∞. Therefore, the initial and two boundary conditions for an infinite plate with negligible convective resistance are:

$$\begin{aligned} X(x,0) &= X_0 \\ X(-L,t) &= X_\infty \\ X(L,t) &= X_\infty \end{aligned} \tag{2}$$

For an infinite plate with negligible convective resistance, Equations (1) and (2), the solution for the average moisture content with a quadratic time behavior for diffusivity is given as (Iribe-Salazar et al., 2018):

$$\frac{\overline{X} - X_\infty}{X_0 - X_\infty} = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\exp\left[-\frac{(2n-1)^2\pi^2\left(D_0 + \frac{1}{2}D_1 t + \frac{1}{3}D_2 t^2\right)t}{4L^2}\right] \tag{3}$$

The dimensionless time τ, is written as:

$$\tau = \frac{\left(D_0 + \frac{1}{2}D_1 t + \frac{1}{3}D_2 t^2\right)t}{L^2} \tag{4}$$

For each time, τ was solved from Equation (3) with Newton's second-order method, using 10 terms in the infinite series. Once the dimensionless time was known, this equation was reordered to:

$$\frac{\tau L^2}{t} = D_0 + \frac{1}{2}D_1 t + \frac{1}{3}D_2 t^2 \tag{5}$$

The D_0, D_1, and D_2 parameters are estimated through linear regression analysis. For determining the mean and standard deviation in the input D_0, D_1, and D_2 parameters, this procedure was performed 40 times, one for each of the mass transfer kinetics.

2.3.2. Theoretical probabilistic model for baking

The theory of functions of random variables is used to obtain estimates for the distribution in average moisture content against time. The governing equation of the system, Equation (3) can be rewritten as:

$$\overline{X} = X_\infty + \frac{8}{\pi^2}(X_0 - X_\infty)\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\exp\left[-(2n-1)^2 rt\right] \tag{6}$$

where the mass rate constant, r is:

$$r = \frac{\pi^2\left(D_0 + \frac{1}{2}D_1 t + \frac{1}{3}D_2 t^2\right)}{4L^2} \tag{7}$$

For the baking model given by Equation (6), the average moisture content of the flat plate is an exponential function of the rate constant, r. Accordingly, the variability of the system defined by Equation (6) is determined by the input variables X_0, X_∞, and r, which can be assumed as independent random variables, governed by probability density functions. It is also assumed that dispersion in these input parameters can be represented by a Normal distribution. These considerations will be validated with Monte Carlo simulations and experimental results. Since $\overline{X}$ is a function of three random variables (X_0, X_∞, r) with a joint density function $f(X_0, X_\infty, r)$, the expected value and the variance of the average moisture content are given by:

$$E\left(\overline{X}\right) = \mu_{\overline{X}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{X}\, f(X_0, X_\infty, r)\, dX_0\, dX_\infty\, dr \tag{8}$$

$$V\left(\overline{X}\right) = {\sigma_{\overline{X}}}^2 = E\left[\left(\overline{X} - \mu_{\overline{X}}\right)^2\right] \tag{9}$$

Since X_0, X_∞, and r are independent, their joint density function is:

$$f(X_0,\ X_\infty,\ r) = f_{X_0}(X_0)\ f_{X_\infty}(X_\infty)\ f_r(r) \tag{10}$$

The mean and variance of the average moisture content between tortillas were estimated for any time during the baking process. If N terms are considered in the solution for the average moisture content ($\overline{X}$), the expected value ($\mu_{\overline{X}}$) is:

$$\mu_{\overline{X}} = \mu_{X_\infty} + \frac{8}{\pi^2}\left(\mu_{X_0} - \mu_{X_\infty}\right)\sum_{n=1}^{N}\frac{1}{(2n-1)^2}\exp\left[-(2n-1)^2\mu_r t + \frac{1}{2}(2n-1)^4\sigma_r^2 t^2\right] \tag{11}$$

The mean and standard deviation of the input random variables should be known: initial moisture content (μ_{X_0}, σ_{X_0}), equilibrium moisture content ($\mu_{X_\infty}, \sigma_{X_\infty}$), and mass rate constant (μ_r, σ_r). The latter depends on the distribution in half-thickness (μ_L, σ_L) and effective diffusivity parameters ($\mu_{D_0}, \sigma_{D_0},\ \mu_{D_1}, \sigma_{D_1},\ \mu_{D_2}, \sigma_{D_2}$). By defining,

$$\mu_D = \mu_{D_0} + \frac{1}{2}\mu_{D_1} t + \frac{1}{3}\mu_{D_2} t^2 \tag{12}$$

then, the mean for the rate constant results in:

$$\mu_r = \frac{\pi^2 \mu_D}{4\mu_L^2} \tag{13}$$

The method of statistical differentials (Kempthorne and Folks, 1971) can be used to estimate the variance in the rate constant:

$$\sigma_r^2 = \left(\frac{\partial r}{\partial D_0}\right)^2 \sigma_{D_0}^2 + \left(\frac{\partial r}{\partial D_1}\right)^2 \sigma_{D_1}^2 + \left(\frac{\partial r}{\partial D_2}\right)^2 \sigma_{D_2}^2 + \left(\frac{\partial r}{\partial L}\right)^2 \sigma_L^2 \tag{14}$$

After obtaining the partial derivatives, the standard deviation in r results in:

$$\sigma_r = \mu_r\sqrt{\left(\frac{\sigma_{D_0}}{\mu_D}\right)^2 + \left(\frac{1}{2}t\frac{\sigma_{D_1}}{\mu_D}\right)^2 + \left(\frac{1}{3}t^2\frac{\sigma_{D_2}}{\mu_D}\right)^2 + \left(2\frac{\sigma_L}{\mu_L}\right)^2} \tag{15}$$

On the other hand, when one term in the solution for the average moisture content ($\overline{X}$) is considered, the variance in the average moisture content (${\sigma_{\overline{X}}}^2$) given by Equation (9) is:

$${\sigma_{\overline{X}}}^2 = \sigma_{X_\infty}^2 + \left[\frac{8}{\pi^2}\left(\mu_{X_0} - \mu_{X_\infty}\right)e^{-\mu_r t + \frac{1}{2}\sigma_r^2 t^2}\right]^2 \left\{\left(1 + \frac{\sigma_{X_0}^2 + \sigma_{X_\infty}^2}{\left(\mu_{X_0} - \mu_{X_\infty}\right)^2}\right)e^{\sigma_r^2 t^2} - 1\right\} \tag{16}$$

In the same way, if two terms in the solution are considered, the variance is:

$${\sigma_{\overline{X}}}^2 = \sigma_{X_\infty}^2 + \left[\frac{8}{\pi^2}\left(\mu_{X_0} - \mu_{X_\infty}\right)e^{-\mu_r t + \frac{1}{2}\sigma_r^2 t^2}\right]^2 \times \left\{\left(1 + \frac{\sigma_{X_0}^2 + \sigma_{X_\infty}^2}{\left(\mu_{X_0} - \mu_{X_\infty}\right)^2}\right)\left(e^{\sigma_r^2 t^2} + \frac{2}{9}e^{-8\mu_r t + 49\sigma_r^2 t^2} + \frac{1}{81}e^{-16\mu_r t + 161\sigma_r^2 t^2}\right) - \left(1 + \frac{1}{9}e^{-8\mu_r t + 40\sigma_r^2 t^2}\right)^2\right\} \tag{17}$$

When N terms are considered, the variance in the average moisture content (${\sigma_{\overline{X}}}^2$) is:

$${\sigma_{\overline{X}}}^2 = \sigma_{X_\infty}^2 + \left[\frac{8}{\pi^2}\left(\mu_{X_0} - \mu_{X_\infty}\right)e^{-\mu_r t + \frac{1}{2}\sigma_r^2 t^2}\right]^2 \times \left\{\left(1 + \frac{\sigma_{X_0}^2 + \sigma_{X_\infty}^2}{\left(\mu_{X_0} - \mu_{X_\infty}\right)^2}\right)\left(\sum_{n=1}^{N}\frac{1}{(2n-1)^4}e^{-8n(n-1)\mu_r t + \left[2(2n-1)^4 - 1\right]\sigma_r^2 t^2} + \right.\right.$$
$$\left.\left. +2\sum_{n=m+1}^{N}\sum_{m=1}^{N-1}\frac{1}{(2n-1)^2(2m-1)^2}e^{-\left[(2n-1)^2 + (2m-1)^2 - 2\right]\mu_r t + \left\{\frac{1}{2}\left[(2n-1)^2 + (2m-1)^2\right]^2 - 1\right\}\sigma_r^2 t^2}\right) + -\left(\sum_{n=1}^{N}\frac{1}{(2n-1)^2}e^{-4n(n-1)\mu_r t + \left\{\frac{1}{2}\left[(2n-1)^4 - 1\right]\right\}\sigma_r^2 t^2}\right)^2\right\} \tag{18}$$

Note these equations show how the standard deviation in average moisture content is a result of distribution in initial and equilibrium moisture content, and dispersion in the mass rate constant; which in turn depends on distribution in half-thickness and effective diffusivity parameters.

2.4. Monte Carlo probabilistic model for baking

To validate the Theoretical Probabilistic model, Monte Carlo simulations were carried out. In this sense, the parameters involved during unstable mass transfer should be considered as random variables governed by probability density functions. Physically, the dispersion in the average moisture content ($\overline{X}$) of the tortilla during baking originates from the variability of the involved parameters: initial (X_0) and equilibrium (X_∞) moisture content, half-thickness (L), and effective diffusivity parameters (D_0, D_1, and D_2). Therefore, the average moisture content during the non-steady state baking process has a functional dependence on the following six process variables:

$$\overline{X} = f(X_0, X_\infty, L, D_0, D_1, D_2) \tag{19}$$

To incorporate process variability, knowledge of the nature of the probability distributions of these random variables is required. To obtain this information, several replicates were made to measure these variables and the respective means and standard deviations were recorded. Also, sampling was carried out to determine the distribution of the initial moisture content, moisture content at the time of greater variability, and final moisture content of the tortilla shells to validate the model (Caro-Corrales, 2002; Cronin and Kearney, 1998).

In the implementation of the Monte Carlo probabilistic model for baking, uniformly distributed random numbers between zero and one were generated. The Box-Muller (1958) transformation was used to originate random numbers representing a standard Normal distribution. These numbers were used to extract values of the random variables from the probability distributions that govern the parameters involved. Each time the model is run, a value is randomly selected from each of the six random variables and a single average moisture content is calculated. After 500 iterations, the average moisture content distribution is obtained. The Monte Carlo method was coded in Visual Basic 6.0 language for each of the stages described above.

2.5. Model validation

To validate the models, the histories of the mean ($\mu_{\overline{X}}$) and standard deviation ($\sigma_{\overline{X}}$) for the average moisture content of the tortilla shells obtained with the Theoretical Probabilistic model were compared with results obtained through Monte Carlo simulations and with experimental results. Forty experimental mass transfer kinetics were performed.

The random variability in the average moisture content of the tortilla shells during the baking process is determined by the magnitudes of the dispersion in the random variables involved: initial (X_0) and equilibrium (X_∞) moisture content, half-thickness (L), and effective diffusivity (D_e). Their relative effect on dispersion in the average moisture content results from the magnitude of their individual standard deviations and the functional relationship between the average moisture content and these variables.

2.6. Sensitivity analysis

A sensitivity analysis of the parameters involved (X_0, X_∞, L, and D_e) was performed to determine which one contributes the most to the variability of the average moisture content of the baked tortilla shells. The procedure consisted of performing a parametric study with the Monte Carlo method in four separate tests to examine which of these variables is dominant in terms of controlling the standard deviation in the average moisture content of the tortilla shells. Within each test, variability was considered to exist in only one of the four input variables, that is, the standard deviation of the other three remaining variables was considered zero. The relative contribution of each input variable to the dispersion in the average moisture content of the tortilla shells was obtained as the ratio of the variance of the average moisture content when a single variable was active and that when all the input variables were active (Caro-Corrales et al., 2002). Such a study allows to investigate the most efficient strategy to reduce the dispersion in the average moisture content of the tortilla shells.

2.7. Design of experiments

Because the baking conditions in the study remained constant, an analysis was carried out using descriptive statistics, which consisted in obtaining the mean (μ) and standard deviation (σ) for each of the process variables (X_0, X_∞, L, and D_e).

3. Results and discussion

3.1. Input random parameters

Table 1 summarizes the mean and standard deviation for the input random parameters: the initial (X_0) and equilibrium (X_∞) moisture content, half-thickness (L), and coefficients for effective diffusivity (D_0, D_1, and D_2). The mean and standard deviation correspond to 40 experimental replicates and 500 Monte Carlo random iterations for each of the parameters. The coefficient of variation (CV) for the experimental initial and equilibrium moisture content, half-thickness, and coefficients (D_0, D_1, and D_2) for effective diffusivity was 1.66%, 17.7%, 3.46%, 45.4%, 13.9%, and 27.9%, respectively. The high variability during the baking process provided by the D_0 coefficient, which represents the initial effective diffusivity, is probably due to the non-uniformity in the composition between tortillas. The mean for initial and equilibrium moisture content, and half-thickness are in accord with that reported by other researches (Braud et al., 2001; Morales-Pérez and Vélez-Ruiz, 2011; Xu and Kerr, 2012). The order of magnitude found in the initial diffusivity D_0 for corn tortillas was similar to that reported by Kayacier and Sing (2004) for baked snacks. These data are required by the theoretical probabilistic solution to estimate mean and standard deviation moisture content histories. In addition, with the experimental data for the parameters, 500 random samplings were run and the mean and standard deviation achieved with these random items were similar to those obtained from the experiments (Table 1). These sampling data are needed as input random parameters in the Monte Carlo method.

Table 1
Mean and standard deviation for the input random parameters.

	Experimental		Monte Carlo	
	μ	σ	μ	σ
X_0 (kg water/kg d.s.)	0.625	0.0104	0.625	0.0100
X_∞ (kg water/kg d.s.)	0.022	0.0039	0.022	0.0038
L (m)	4.74×10^{-04}	1.64×10^{-05}	4.74×10^{-04}	1.68×10^{-05}
D_0 (m^2/s)	2.72×10^{-11}	1.23×10^{-11}	2.70×10^{-11}	1.24×10^{-11}
D_1 (m^2/s^2)	4.57×10^{-12}	6.34×10^{-13}	4.55×10^{-12}	6.04×10^{-13}
D_2 (m^2/s^3)	-4.55×10^{-15}	1.27×10^{-15}	$- 4.55 \times 10^{-15}$	1.27×10^{-15}

Initial (X_0) and equilibrium (X_∞) moisture content, half-thickness (L), and coefficients for effective diffusivity (D_0, D_1, and D_2). Mean and standard deviation correspond to 40 experimental trials and 500 samplings for each parameter.

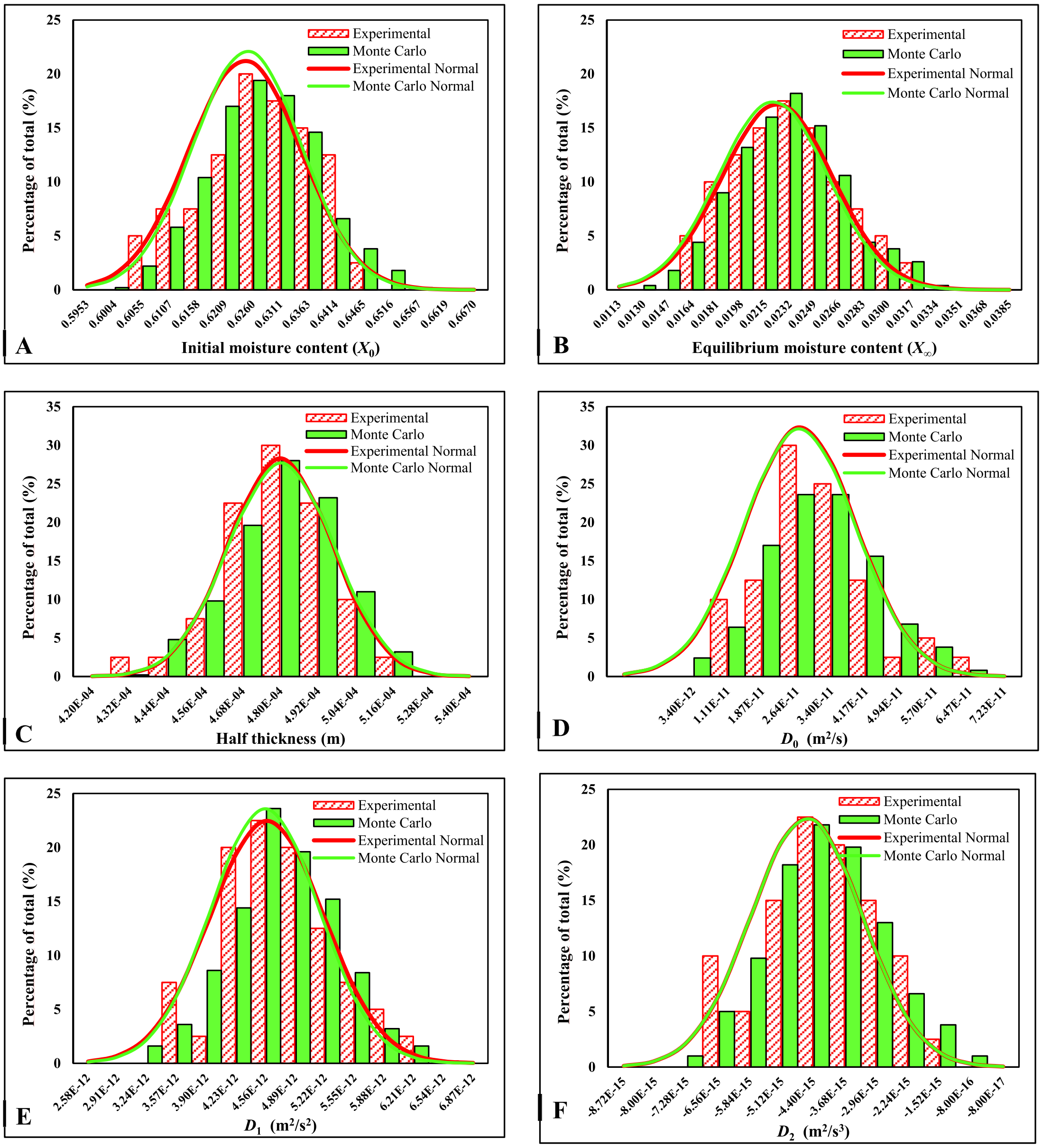

Fig. 1. Distribution in the input random parameters X_0 (A), X_∞ (B), L (C), D_0 (D), D_1 (E), and D_2 (F) with the associated Normal distribution curve. Histograms correspond to 40 experimental replicates and 500 random trials for the Monte Carlo method.

Fig. 1 displays the distribution in X_0, X_∞, L, D_0, D_1, and D_2, in frequency histogram form along with the associated Normal distribution curve, as found by experiments (40 replicates) and as sampled for the Monte Carlo technique (500 random trials). The histograms resemble a bell-shaped curve and seem to be no tendency in displacement in each of the input variables, similar intervals, and upper and lower limits were obtained, in both, the random sampling and experimental instances. This allows inferring that the distributions display Normal-type behavior. To verify that the obtained data for each of the parameters can be adequately described by a Normal distribution, three statistical tests (Shapiro-Wilk, Kolmogorov-Smirnov, and Anderson-Darling) were carried out. They indicated that the idea that data came from a Normal distribution cannot be rejected ($p \geq 0.05$) with 95% confidence.

3.2. Predictions and validation of the theoretical probabilistic model

Fig. 2 shows the mean (μ) and standard deviation (σ) histories for the average moisture content as predicted by the Theoretical Probabilistic model, and their comparison with the Monte Carlo method and experimental results. In the estimation of the average moisture content, using four terms in the series solution, the error is less than 0.1% in the unaccomplished moisture ratio $(\overline{X} - X_\infty)/(\overline{X} - X_\infty)$ for mass transfer Fourier numbers greater than 0.013 ($t > 20.7$ s). The mean for the average moisture content ($\mu_{\overline{X}}$) estimated by the Theoretical Probabilistic model practically superimposed to those estimated by Monte Carlo ($R^2 = 0.9998$) and from the experiments ($R^2 = 0.9998$) throughout the baking process. Moreover, the theoretical probabilistic solution adequately predicts the standard deviation for the average moisture content ($\sigma_{\overline{X}}$) throughout the baking process. The standard deviation history showed an adequate fit to those estimated by Monte Carlo ($R^2 = 0.9950$) and obtained by experiments ($R^2 = 0.9800$). This indicates that the considerations made for the development of the Theoretical Probabilistic approach were adequate.

Results indicated that the standard deviation for the average moisture content ($\sigma_{\overline{X}}$) as predicted by the Theoretical Probabilistic model and by Monte Carlo rises from initial moisture content, reaches a maximum, and then, falls back until reaching an asymptotic behavior. This arises

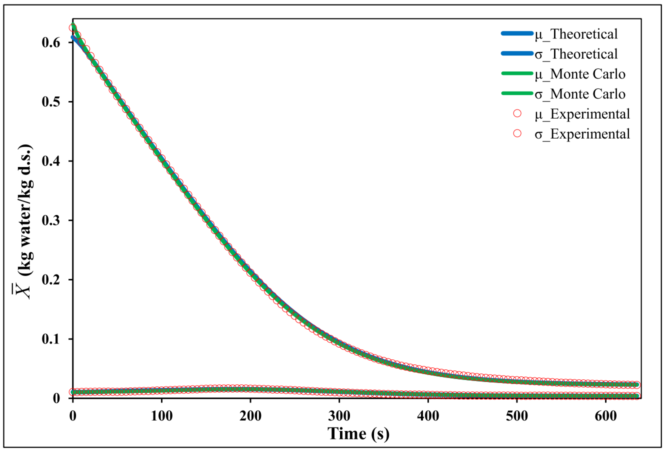

Fig. 2. Mean (μ) and standard deviation (σ) histories. Theoretical-Monte Carlo (mean $R^2 = 0.9998$; std. dev. $R^2 = 0.9950$), Theoretical-experimental (mean $R^2 = 0.9998$; std. dev. $R^2 = 0.9800$).

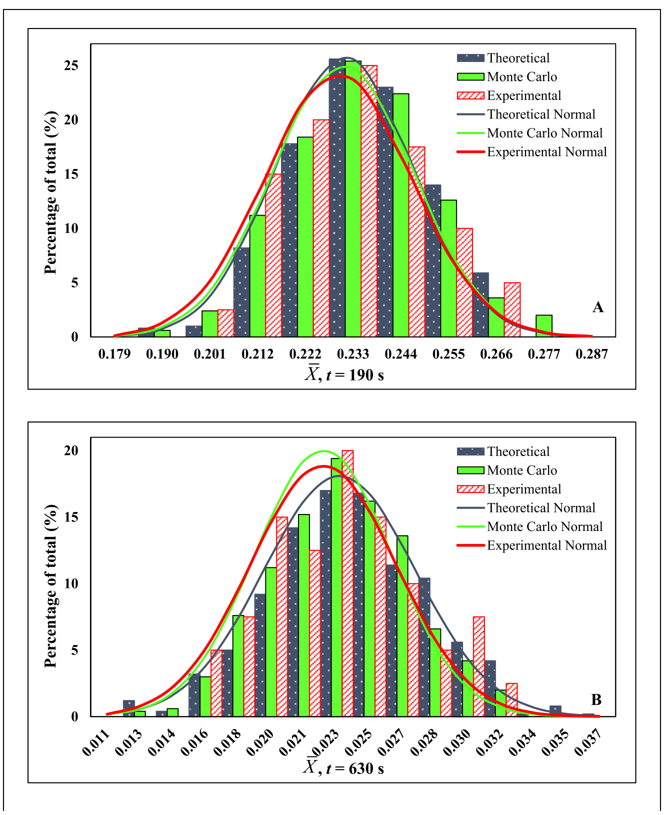

Fig. 3. Distribution in the average moisture content ($\overline{X}$) with the associated Normal distribution curve at 190 s (A) and 630 s (B).

because there is a sufficiently long baking time (630 s) for the moisture content of all the tortilla shells to reach the equilibrium moisture content. Results obtained for the mean of average moisture content ($\mu_{\overline{X}}$), as well as for standard deviation ($\sigma_{\overline{X}}$), are essentially an assertion of the correctness of the chosen deterministic model and the proper description of the input random parameters for both models.

Fig. 3 displays the distribution in the average moisture content ($\overline{X}$), in frequency histogram form as well as the corresponding Normal distribution curve, as predicted by the Theoretical Probabilistic model, the Monte Carlo method, and found from the experiments, at 190 s and 630 s. The highest variability in the average moisture content was at 190 s (Fig. 3A). Despite that dispersion, data show a Normal-type behavior ($p \geq 0.05$), indicated by the three aforementioned statistical tests. The target moisture content for baked tortilla shells is around 0.02 kg water/kg d. s; therefore, the distribution in average moisture content at 630 s is shown in Fig. 3B. Data results indicated that it comes from a Normal distribution with 95% confidence in the three performed statistical tests. Among the research conducted about obtaining output variables with a Normal-type behavior, the following stand out: Cronin et al. (2000) studied the thermal processing of discrete solid products and outlined the development of a theoretical framework to allow the effects of random variability in the thermal processing of solids to be incorporated into process design studies, Caro-Corrales et al. (2002) analyzed the random variability in biscuit cooling and declared the outlined methodology is capable of being applied to industrial use, Hong et al. (2016) studied the inactivation of *Clostridium botulinum* spores in beef meatball trays and found the four most critical factors affecting the accumulation of lethality, and Cronin and Kearney (1998) applied the Monte Carlo method to quantify and optimize the final distribution in moisture content for drying of vegetables. These researchers described that their analyzed variables showed a Normal-type behavior.

It is interesting to emphasize that the Theoretical Probabilistic model and the Monte Carlo method did not show any tendency to predict a mean or a distribution much higher or lower than it occurs in practice. Both, the Theoretical Probabilistic model and the Monte Carlo method, for the tortilla shells baking process, estimate the average moisture content mean along with its variability; as well as, they allow to analyze the limits of the distribution and the proportion of the product lying within any moisture content band, helping to assess the quality of the product. This information is important in the context of process optimization.

The mean and standard deviation for the average moisture content as predicted by the Theoretical Probabilistic model, the Monte Carlo method, and obtained from experimental results at 0, 190, and 630 s are given in Table 2. Estimations generated for the mean of the average moisture content with the Theoretical Probabilistic model are similar to those predicted by the Monte Carlo method and obtained from experimental data.

3.3. Sensitivity analysis

Random variability in average moisture content at the termination of the baking process is determined by the magnitude of the variability in the input random parameters. Their relative effect on the dispersion in the moisture content is a result of the magnitude of their standard deviations and the functional relationship between these parameters and average moisture content. Therefore, the sensitivity analysis at the termination of the baking process using the Monte Carlo method (Table 3) indicated that variations in certain parameters have a significant effect on the required average moisture content and, therefore, on the design of the baking process. These parameters were classified from highest to lowest influence based on the observed effects within the ranges analyzed as follows: equilibrium moisture content (X_∞), half-thickness (L), effective diffusivity (D_e), and initial moisture content (X_0). The standard deviation when all the variables were active was

Table 2
Mean (μ) and standard deviation (σ) for ($\overline{X}$) estimated by the Theoretical Probabilistic model, the Monte Carlo method, and from experiments.

Average Moisture Content (kg water/kg d. s.)						
Time (s)	Theoretical solution		Monte Carlo output		Experimental results	
	μ	σ	μ	σ	μ	σ
0	0.600	0.0107	0.616	0.0103	0.625	0.0104
190	0.235	0.0154	0.235	0.0155	0.229	0.0165
630	0.023	0.0039	0.024	0.0039	0.022	0.0039

Table 3
Sensitivity analysis at the end of the baking process (630 s).

Active parameter	Standard deviation	Standard deviation in $\overline{X}$	$\sigma^2_{\text{parameter}}/\sigma^2_{\text{total}}$	Relative contribution (%)
X_0 (kg water/kg d.s.)	0.011	1.63×10^{-05}	1.86×10^{-05}	0.002
X_∞ (kg water/kg d.s.)	0.004	3.75×10^{-03}	0.9853	98.53
$D = D_0 + ½D_1t + {}^1/_3D_2t^2$ (m^2/s)	2.36×10^{-11}	1.55×10^{-04}	0.0017	0.17
L (m)	1.63×10^{-05}	4.29×10^{-04}	0.0129	1.29

0.0038 kg water/kg d. s. The analysis indicated that the equilibrium moisture content caused the greatest dispersion in average moisture content with 98.53% of the total variance. The variability in semi-thickness (L) and effective diffusivity (D_e) had little contribution (1.29 and 0.17%), while the dispersion in the initial moisture content (0.002%) contributed the least to random variability in final average moisture content. Due to effective diffusivity and initial moisture content did not substantially affect the variability, it is important to highlight that the composition between tortilla shells was practically homogeneous. Equilibrium moisture content had the greatest contribution to the final moisture content distribution and it is a result of changes in airspeed, temperature, and humidity; therefore, greater control should be exercised on them. In addition, the baking time was long enough for the other parameters to influence the final moisture content, due to the functional relationship between them. Braud et al. (2001) carried out a sensitivity analysis during corn tortilla drying, the involved parameters were: product thickness, drying temperature, convective heat transfer coefficient, and initial moisture content. Their analysis indicated that thickness had a greater effect on the variability of the process related to the initial moisture content; results that coincide with those of the present research.

Although in the initial description of the distribution for the six parameters (X_0, X_∞, L, D_0, D_1, and D_2), at the beginning of the process the D_0 parameter had the greatest variability (45.4%), the sensitivity analysis did not reflect it in the effective diffusivity (D_e), since this parameter had very little contribution (0.17%) in random variability of the average moisture content at the termination of the baking process. However, in the description of the parameters, initial moisture content, X_0 showed the lowest variability (1.66%) and in the sensitivity analysis this parameter affected with the lowest percentage (0.002%) of the total variance in moisture content. This indicates that the relative effect of the input random parameters does not depend solely on the magnitude of their dispersion, but also on the functional relationship between them and the average moisture content. On the other hand, the variations of the evaluated parameters were within the typical operating conditions that can exist in the production of baked corn tortilla shells.

Knowing the parameters that significantly affect the baking of corn tortilla shells is useful in design operations to determine which conditions need more sensitive control and to assess how controlling the uncertainty in these parameters can reduce variations in the average moisture content, when deciding on the most efficient strategy to decrease dispersion in the final product. The methodology described in this research can be applied to the industrial process of baked tortilla shells production to achieve a more uniform product in terms of the variability that occurs in average moisture content; since one of the requirements is that the baked tortilla be within a specific band of moisture content at the end of the process, and therefore, achieve a decrease in the amount of defective product, reducing processing costs, which will benefit the producer and exporter of baked tortilla shells.

4. Conclusions

The lowest variability in the average moisture content ($\sigma_{\overrightarrow{X}} =$ 0.0039 kg water/kg d. s.) occurred at 630 s and the greatest variation ($\sigma_{\overrightarrow{X}} = 0.0165$ kg water/kg d. s.) at 190 s. The probability distributions for initial (X_0) and equilibrium (X_∞) moisture content, half-thickness (L), and effective diffusivity with a quadratic function of time (D_0, D_1, and D_2) showed a Normal behavior with mean and standard deviation of 0.625 ± 0.0104 kg water/kg d. s., 0.022 ± 0.0039 kg water/kg d. s., $4.74 \times 10^{-4} \pm 1.64 \times 10^{-5}$ m, $2.72 \times 10^{-11} \pm 1.23 \times 10^{-11}$ m^2/s, $4.57 \times 10^{-12} \pm 6.34 \times 10^{-13}$ m^2/s^2, $-4.55 \times 10^{-15} \pm 1.27 \times 10^{-15}$ m^2/s^3, respectively.

The estimates for the mean and standard deviation of the average moisture content with the Theoretical Probabilistic model satisfactorily fit those obtained with Monte Carlo simulations ($R^2 = 0.9998$ and $R^2 = 0.9950$) and experiments ($R^2 = 0.9998$ and $R^2 = 0.9800$), during the entire baking process of tortilla shells. These results reflect the appropriate deterministic model used and the correct characterization of the input random variables. Furthermore, it indicates that considerations made during the development of the Theoretical Probabilistic model were appropriate.

The sensitivity analysis with the Monte Carlo method indicated that at the end of the baking process (630 s), equilibrium moisture content caused the greatest dispersion (98.53%) while the initial moisture content contributed the least (0.002%) to the variability in the moisture content of baked tortilla shells. To reduce variability in the final moisture content, it is suggested to achieve greater control over the conditions of the baking fluid: speed, temperature, and humidity of the air.

The influence of different baking conditions on the variability in the final average moisture content of tortilla shells should be analyzed as an aid to select the optimal baking conditions. The parameters that produced the greatest contribution to the dispersion in moisture could be identified, which will allow the use of a more efficient strategy that reduces the variability in the moisture content of the product under study.

CRediT authorship contribution statement

Rosalina Iribe-Salazar: Conceptualization, Methodology, Data curation, Writing - original draft. **José Caro-Corrales:** Conceptualization, Methodology, Data curation, Writing - original draft. **Yessica Vázquez-López:** Supervision, Software, Validation, Writing - review & editing.

Declaration of competing interest

The authors state that they have no conflict of interests or personal relationships that could influence the work reported in this study.

Acknowledgments

This research was supported by Programa de Fomento y Apoyo a Proyectos de Investigación (PROFAPI-2015/059), Universidad Autónoma de Sinaloa.

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